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DISTRIBUTION OF ELECTRONS IN THE FIELD OF A
CHARGED SPHERE EMITTING ELECTRONS

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DISTRIBUTION OF ELECTRONS IN THE FIELD OF A
CHARGED SPHERE EMITTING ELECTRONS

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ABSTRACT: The density of the electron number in the field of a small spherical particle is calculated for two cases: (1) where the number of electrons is infinite, and (2) where the number of electrons is finite.

Spherical particles of small radius which thermally emit electrons may exist in gases in the form of a high-temperature admixture. A calculation of the conductivity of such dust gases is of great interest. The conductivity can be computed with an unknown potential field created by the particle charge and the electrons and particles surrounding it [1]. /3*

We take as given the charge of the particle Ze which has a radius R^n , and the density of the electron number at infinity $n_{e\infty}$. We also assume that the field around a particle in a spherical volume equal to the mean volume per particle in the gas is determined only by the electrons in this volume and the particle charge. The latter assumption, obviously, is satisfied only with high particle charges and low densities of the particle number.

Let us examine the case when the mean free path length of the electrons is significantly greater than the activity region of the field surrounding the particles.

The potential can be calculated by the self-consistent field method using the Poisson equation

$$\Delta U = 4\pi en_e, \quad (1)$$

where U is the spherically-symmetrical potential and n_e is the density of the electron number, which is a function of the distance r to the center of the particle and of the potential U .

This paper is devoted to a calculation of the density of the electron number n_e near the particle. Later we will show that n_e may be computed in explicit form with an unknown type of potential. Our calculations will be made for coulomb type potential

$$U(r) = -\frac{Ze^2}{r}. \quad (2)$$

(1) Let us now compute the density of the number of infinite

* Numbers in the margin indicate pagination in the foreign text.

electrons n_e^{inf} ; to do this we must know the function of electron distribution by velocity in the region near the particle where this function must satisfy the kinetic equation with a collision term equal to zero. We know that in this case the distribution function is an arbitrary function of the motion integrals; i.e., of the total energy and moment of momentum [2]. The precise form of the function is determined by the boundary conditions. The boundary conditions for electrons relating to the activity sphere of the particle field from infinity is the Maxwell distribution function

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$$f_{\text{dif}} = n_{e\infty} \left(\frac{m}{2\pi k T_e} \right)^{3/2} e^{-\frac{mv^2}{2kT_e}}, \quad (3)$$

where T_e is the electron temperature at infinity; m is the electron mass; k is the Boltzmann constant; v is the electron velocity at infinity.

In the activity region of the field, the energy in the exponent of the exponential function is replaced by

$$\epsilon = \frac{mv^2}{2} + U(r). \quad (4)$$

The velocity vector of the electron V remains in one plane when the electron is moving in the particle field. Expressing V in polar coordinates and taking (4) into account, we can present (3) in the form

$$f_{\text{dif}} = n_{e\infty} \left(\frac{m}{2\pi k T_e} \right)^{3/2} e^{-\frac{m}{2kT_e} (v_r^2 + v_\theta^2) - \frac{U(r)}{kT_e}}, \quad (5)$$

where v_r is the radial velocity component and v_θ is the tangential velocity component; the origin is at the particle center.

Function (5) is distorted by the presence of an absorbing surface with radius R .

In (5) the infinite electrons which reach this surface have

$$v_r \leq 0. \quad (6)$$

The total energy ϵ of the infinite electrons satisfies the condition $\epsilon \geq 0$ equivalent to the relationship

$$\frac{mv_r^2}{2} + \frac{mv_\theta^2}{2} > -U(r), \quad (7)$$

Function (5) also describes the velocity distribution of the infinite electrons which travel from infinity yet do not reach the particle surface. For these electrons, the following relationships hold:

$$-\infty \leq v_r \leq \infty, \quad (8)$$

$$\frac{mv_r^2}{2} + \frac{mv_\theta^2}{2} \geq -U(r), \quad (9)$$

$$\frac{mv_r^2}{2} + \frac{mv_\theta^2(r)}{2} + U(r) \leq \frac{mv_\theta^2(R)}{2} + U(R). \quad (10)$$

The infinite electrons, which are formed due to the thermal emission from the surface of the particle according to [3], have the following distribution on the particle surface:

$$f_{em} = \frac{2m^3}{h^3} \left[1 + e^{\frac{1}{kT_p} \left(\frac{mU^2}{2} + W_a \right)} \right]^{-1}, \quad (11)$$

where h is the Planck constant; T_p is the particle temperature; W_a is the work of electron exit from the particle; u is the velocity of the electron near the surface of the particle.

Usually, $W_a \geq (3-4)kT_p$; therefore (11) becomes

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$$f_{em} = \frac{2m^3}{h^3} e^{-\frac{1}{kT_p} \left(\frac{mU^2}{2} + W_a \right)}. \quad (12)$$

By converting in (12) to velocity v , we obtain

$$f_{em} = \frac{2m^3}{h^3} e^{-\frac{1}{kT_p} \left[\frac{mv^2}{2} + U(r) \right] - \frac{1}{kT_p} [W_a - U(R)]}. \quad (13)$$

Function (13) describes the velocity distribution of the infinite electrons under the conditions

$$v_r \geq 0, \quad (14)$$

$$\frac{mv_r^2}{2} + \frac{mv_\theta^2}{2} \geq -U(r), \quad (15)$$

$$\frac{mv_r^2}{2} + \frac{mv_\theta^2(r)}{2} + U(r) \geq \frac{mv_\theta^2(R)}{2} + U(R). \quad (16)$$

With the help of the expressions obtained for the distribution function of the infinite electrons, we can compute by integration the density of the number of infinite electrons in the vicinity of the particle

$$n_i^{(inf)} = 2\pi \int_{v_r \leq 0} f_{dif} v_\theta dv_\theta dv_r + \int_{v_r^2 + v_\theta^2 \geq -\frac{2}{m} U(r)} \quad (17)$$

$$\begin{aligned}
& + 2\pi \int_{v_r \geq 0}^{\infty} f_{\text{dif}} v_{\theta} dv_{\theta} dv_r + \\
& \quad v_r^2 + v_{\theta}^2 \geq -\frac{2}{m} U(r) \\
& \quad v_{\theta}^2 \left(\frac{r^2}{R^2} - 1 \right) - v_r^2 \geq \frac{2}{m} [U(r) - U(R)] \\
& + 2\pi \int_{v_r \geq 0}^{\infty} f_{\text{em}} v_{\theta} dv_{\theta} dv_r, \\
& \quad v_r^2 + v_{\theta}^2 \geq -\frac{2}{m} U(r) \\
& \quad v_{\theta}^2 \left(\frac{r^2}{R^2} - 1 \right) - v_r^2 \leq \frac{2}{m} [U(r) - U(R)]
\end{aligned} \tag{17}$$

When determining the integration limits in the second and third integrals in expression (17), conditions (10) and (16) were transformed. As a result, we find that

$$\frac{mv_{\theta}^2(r)}{2} = \frac{M^2}{2mr^2}, \tag{18}$$

$$\frac{mv_{\theta}^2(R)}{2} = \frac{M^2}{2mR^2}, \tag{19}$$

where M is the moment of momentum.

The value $M = \text{const}$ for an electron in a particle field without collisions. Thus, for example, instead of (10), using relationships (18) and (19) we obtain

$$\frac{mv_r^2}{2} + \frac{M^2}{2mr^2} \left(1 - \frac{r^2}{R^2} \right) \leq -[U(r) - U(R)]. \tag{20}$$

Using (18) we find that

$$\frac{mv_{\theta}^2}{2} \left(\frac{r^2}{R^2} - 1 \right) - \frac{mv_r^2}{2} > U(r) - U(R). \tag{21}$$

By integrating (17) we obtain

$$\begin{aligned}
n_e^{(\text{inf})} = n_{\infty} & \left\{ \frac{1}{\sqrt{\pi}} \sqrt{-\frac{U(r)}{kT_e}} \left[1 + \sqrt{1 - \frac{R^2 U(R)}{r^2 U(r)}} \right] + \right. \\
& + \frac{1}{2} e^{-\frac{U(r)}{kT_e}} \left[1 - \Phi \left(\sqrt{-\frac{U(r)}{kT_e}} \right) \right] + \\
& + \frac{1}{2} \sqrt{1 - \frac{R^2}{r^2}} e^{-\frac{[r^2 U(r) - R^2 U(R)]}{kT_e (r^2 - R^2)}} \left[1 - \Phi \left(\sqrt{-\frac{r^2 U(r) - R^2 U(R)}{kT_e (r^2 - R^2)}} \right) \right] \right\} + \\
& + 2 \left(\frac{2\pi m k T_p}{h^3} \right)^{1/2} e^{-\frac{1}{kT_p} [W_0 - U(R)]} \left\{ \frac{1}{\sqrt{\pi}} \sqrt{-\frac{U(r)}{kT_p}} \left(1 - \sqrt{1 - \frac{R^2 U(R)}{r^2 U(r)}} \right) + \right. \\
& \left. + \frac{1}{2} e^{-\frac{U(r)}{kT_p}} \left[1 - \Phi \left(\sqrt{-\frac{U(r)}{kT_p}} \right) \right] \right\}
\end{aligned} \tag{22}$$

$$-\frac{1}{2} \sqrt{1 - \frac{R^2}{r^2}} e^{-\frac{[r^2 U(r) - R^2 U(R)]}{kT_p(r^2 - R^2)}} \left[1 - \Phi \left(\sqrt{-\frac{r^2 U(r) - R^2 U(R)}{kT_p(r^2 - R^2)}} \right) \right]. \quad (22)$$

The expression in the first set of figured brackets of (22) and the factor $n_{e\infty}$ are obtained as a result of computing the first two integrals of (17). All the remaining terms on the right-hand side of (22) are obtained by computing the last integral in (17). In (22), the value $\Phi(x)$ is a probability integral.

(2) Let us turn now to calculation of the density of the number of finite electrons. Near the particles which emit the electrons there are only two possible types of finite electrons. The orbits of one type of electron intersect the particle surface; the orbits of the second type do not intersect it. Electrons with orbits of the first type are those electrons which are emitted thermally by a particle and whose total energy is $\epsilon \leq 0$.

The distribution function of such electrons coincides with (13), but we should here assume that $\epsilon \leq 0$.

Finite electrons of the second type can be formed as a result of infrequent collisions of infinite electrons and finite electrons, having orbits passing through the particle, with neutral atoms of the gas or with other electrons. In collisions with electrons, the major process of electron transfer to finite orbits which do not pass through the particle is the process of electron deceleration due to distant interactions.

For each value $\epsilon < 0$, the boundary distribution function for finite electrons with orbits which do not pass through the particle is distribution function (13) with $\epsilon < 0$. When $\epsilon = 0$ there are two boundary functions, (5) and (13), for the distribution function. It is obvious that when $\epsilon = 0$ the distribution function will be

$$f = n_{e\infty} \left(\frac{m}{2\pi kT_e} \right)^{3/2} + \frac{2m^3}{h^3} e^{-\frac{1}{kT_p} [W_d - U(R)]}. \quad (23)$$

For small deviations from $\epsilon = 0$ (when $\epsilon < 0$) the distribution function will be a combination of (13) and a certain function f_0 , /7 whose role increases as the deviation of the energy from zero increases.

Subsequently, we will ignore the small region in which the distribution function is a combination of (5) and (13) assuming that the distribution function of finite electrons whose orbits pass through a particle has the form of (13) when $\epsilon \leq 0$.

Now it is easy to compute the density of the number of finite electrons

$$n_e^{(fin)} = 2\pi \int_{v_r^2 + v_\theta^2 \leq \frac{1}{m} U(r)} f_{em} v_\theta dv_\theta dv_r. \quad (24)$$

From (24) we find that

$$n_e^{(fin)} = 2 \left(\frac{2\pi m k T_p}{h^3} \right)^{1/2} e^{-\frac{1}{k T_p} [W_a - U(R)]} \left[-\frac{2}{\sqrt{\pi}} \sqrt{-\frac{U(r)}{k T_p}} + e^{-\frac{U(r)}{k T_p}} \Phi \left(\sqrt{-\frac{U(r)}{k T_p}} \right) \right]. \quad (25)$$

From (22) and (25) we obtain the following expression for the density of the electron number in the field of a positively-charged emitting sphere:

$$\begin{aligned} n_e = n_{eco} & \left\{ \frac{1}{\sqrt{\pi}} \sqrt{-\frac{U(r)}{k T_e}} \left[1 + \sqrt{1 - \frac{R^2 U(R)}{r^2 U(r)}} \right] + \right. \\ & + \frac{1}{2} e^{-\frac{U(r)}{k T_e}} \left[1 - \Phi \left(\sqrt{-\frac{U(r)}{k T_e}} \right) \right] + \\ & + \frac{1}{2} \sqrt{1 - \frac{R^2}{r^2}} e^{-\frac{r^2 U(r) - R^2 U(R)}{k T_e (r^2 - R^2)}} \left[1 - \Phi \left(\sqrt{-\frac{r^2 U(r) - R^2 U(R)}{k T_e (r^2 - R^2)}} \right) \right] \Big\} + \\ & + 2 \left(\frac{2\pi m k T_p}{h^3} \right)^{1/2} e^{-\frac{1}{k T_p} [W_a - U(R)]} \left\{ -\frac{1}{\sqrt{\pi}} \sqrt{-\frac{U(r)}{k T_p}} \left(1 + \sqrt{1 - \frac{R^2 U(R)}{r^2 U(r)}} \right) + \right. \\ & + \frac{1}{2} e^{-\frac{U(r)}{k T_p}} \left[1 + \Phi \left(\sqrt{-\frac{U(r)}{k T_p}} \right) \right] - \\ & \left. - \frac{1}{2} \sqrt{1 - \frac{R^2}{r^2}} e^{-\frac{r^2 U(r) - R^2 U(R)}{k T_p (r^2 - R^2)}} \left[1 - \Phi \left(\sqrt{-\frac{r^2 U(r) - R^2 U(R)}{k T_p (r^2 - R^2)}} \right) \right] \right\}. \end{aligned} \quad (26)$$

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